Fluctuations in HIC and the structure of QCD phase diagram.

Vladimir Skokov

Western Michigan University

June 18, 2014

Outline

- Fluctuations in A-A collisions: Initial or final state fluctuations?
- Static thermal equilibrium predictions (and predictions for observables)
  - small chemical potential, remnants of $O(4)$-scaling
  - critical point: $Z(2)$ scaling, observables
  - first order phase transition: spinodal decomposition
  - quarkyonic phase
  - are the signal unique?
- Beyond statics: dynamical critical phenomena
  - H-model for liquid-gas phase transition
  - spinodal decomposition, statics and dynamics.

- Initial state fluctuations. Can be tested in pp collisions. Distribution of particles: negative binomial

$$P^{\mathrm{NB}}(n) = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{k^k \langle n \rangle^n}{(k+\langle n \rangle)^{n+k}}$$

with the cumulants

$$c_2 = \langle n \rangle \left(1 + \frac{\langle n \rangle}{k}\right), \quad c_4 = c_2 \left(1 + 6\frac{\langle n \rangle}{k}\left[1 + \frac{\langle n \rangle}{k}\right]\right).$$

  - Negative binomial distribution was derived in the CGC framework (Gelis, Lappi, McLerran 2009): $k \propto \frac{N_c^2-1}{2\pi} Q_s^2 S_\perp$
  - Net baryon (or net proton distribution):

$$P(n_{\mathrm{net}}) = \sum_{n_b=1}^{\infty} \sum_{n_{\bar{b}}=1}^{\infty} \delta_{n_{\mathrm{net}}, n_b - n_{\bar{b}}} P^{\mathrm{NB}}(n_b) P^{\mathrm{NB}}(n_{\bar{b}})$$

Negative binomial

- n-th cumulant of net baryon distribution

$$c_n^{\text{net}} = c_n^b + (-1)^n c_n^{\bar{b}}$$

- Zero chemical potential: cumulants of baryon and anti-baryon coincide, thus we can use the results for cumulants of a single negative binomial to derive

$$\frac{c_4}{c_2} = \left(1 + 6\frac{\langle n_b \rangle}{k}\left[1 + \frac{\langle n_b \rangle}{k}\right]\right).$$

- Large chemical potentail: cumulants of net baryon distribution are defined by baryons only, so we again get the same result
- For any positive k, we obtain $c_4/c_2 = 1 +$ positive number > 1
- In experiment tails of net-proton distribution are narrower comparing to Poisson with $c_4/c_2 = 1$.
- Initial state fluctuations are washed out by the collective evolution.

Theory predictions

- First principles: Lattice QCD. $\mu = 0$ plus the application of Taylor series in μ_B and μ_S allows to study a larger domain of the phase diagram (within the radius of convergence)

$$p(T,\mu)/T^4 \approx \sum_{i=0}^{n} \frac{1}{i!} \frac{\partial^i (p/T^4)}{\partial(\mu/T)^i}\bigg|_{\mu=0} \left(\frac{\mu}{T}\right)^i$$

- Universality: many properties of a system close to a continuous phase transition are largely independent of the microscopic details of the interaction. Relatively small number of classes, characterized by global features such as the symmetries of the underlying theory, the number of spatial dimensions…This is for static universality. Dynamic universality is more involved.
  Instead of solving QCD at a critical point, one solves a simpler problem. Under an assumption of the CP existence, the universality predicts properties of some observables close to CP.

$$f_s(t,h) = b^{-d} f_s(b^{y_\tau}\tau, b^{y_h}h), \quad y_\tau = \frac{d}{2-\alpha}, \quad y_h = y_\tau \delta\beta$$

- Models: NJL, linear sigma model, matrix models and etc. Trying to capture the essential features of QCD.

Theory predictions for a system at static/close to equilibrium

- Zero chemical potential
- Finite chemical potential
- Critical end point
- Spinodals
- Quarkionic phase

Zero chemical potential: chiral properties I

- The existence of the crossover phase transition is established by lattice QCD
- Boring part of the phase diagram?
- Not really! Two interesting phenomena: chiral and deconfinement phase transitions.
  - Chiral phase transition: underlying second-order transition in the limit of zero pion mass can be probed with high order cumulants.
    At zero pion mass, the pressure has two contributions: analytic and singular.
    $$p(\tau, h) = p_{\text{nonsing.}}(\tau) + p_{\text{sing}}(\tau)$$
    Singular contribution to pressure is usually tiny:
    $$p_{\text{sing}} \sim (-\tau)^{2-\alpha}$$
    Here α is the specific heat critical exponent. For the O(4) universality class $\alpha \approx -0.21$. The scaling variable $\tau = (T - T_c)/T_c + \kappa\mu^2/T_c^2$. At zero chemical potential, taking two derivatives wrt μ are equivalent to taking one derivative wrt T.
  - Taking a derivative w.r.t. τ (i.e. w.r.t. to T) enhances contribution from the singular part. Three derivatives w.r.t. to τ or 6-th derivatives w.r.t. μ:
    $$\frac{\partial^3 p}{\partial \tau^3} \sim \frac{\partial^6 p}{\partial \mu^6} \sim \tau^{-1-\alpha}.$$
    Diverges close to the transition ($\tau \to 0$).

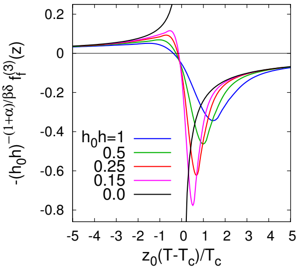
Zero chemical potential: chiral properties II

- Cumulants with order of 6 and higher are sensitive to chiral phase transition.
- For finite pion mass: no divergence. Singular part of pressure is given by

$$p = -b^{-d} f(\tau b^{y_\tau}, h b^{y_h}) \stackrel{(b \to h^{-1/y_h})}{=} -h^{\frac{2-\alpha}{\delta\beta}} f(z), \quad z = \tau h^{-\frac{1}{\delta\beta}}.$$

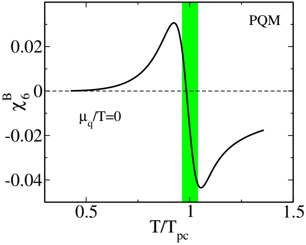
- $$c_6 \propto \partial^6 p / \partial \mu^6 \propto -h^{(-1-\alpha)/\delta\beta} f^{(3)}(z)$$

O(4) scaling function



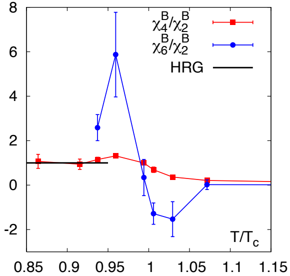
B. Friman et al, 1103.3511

PQM Model



B. Friman et al, 1103.3511

Lattice QCD



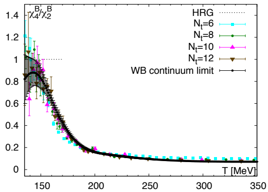
C. Schmidt, 1007.5164, p4

Change of the degrees of freedom:

- Small temperature: degrees of freedom carrying baryon charge have unit baryon number.
$$R_{4,2} = \frac{c_4}{c_2} = \frac{\chi_4}{\chi_2} \to 1$$
- Large temperatures: degrees of freedom carrying baryon charge have fractional baryon number (1/3)
$$R_{4,2} \to 1/9$$
in fact $R_{4,2} \to \frac{1}{9} \cdot \frac{6}{\pi^2}$ owing to Fermi statistics.

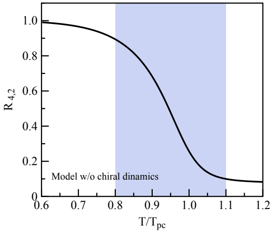
The change from 1 to 1/9 is attributed to deconfinement.

Lattice QCD



S. Borsanyi et al, 1305.5161

Matrix model with quark d.o.f.



Finite chemical potential

We established that deconfinement and chiral transitions lead to negative $c_6(\mu = 0)$.
This already tells us something about fluctuations at finite chemical potential

$$c_4(\mu) = c_4(\mu = 0) + \frac{1}{2} c_6(\mu = 0) \cdot \left(\frac{\mu}{T}\right)^2 + \mathcal{O}\left[\left(\frac{\mu}{T}\right)^4\right]$$

The second term brings negative contribution and dominates for chemical potential

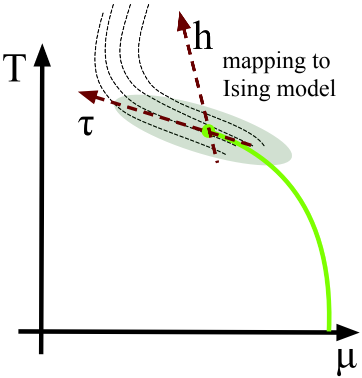
$$\left(\frac{\mu}{T}\right)^2 \geq -\frac{1}{2} \frac{c_6(0)}{c_4(0)}$$

An estimate: if $\frac{c_6(0)}{c_4(0)} \approx -1$, $\frac{\mu}{T} \geq 0.7$ (this corresponds to $\sqrt{s} \approx 40$ GeV).
A decreasing or even negative kurtosis owing to physics of zero chemical potential.

Assuming that CP exists, properties can be predicted.

- QCD CEP: Z(2) static universality class, H-model of dynamic universality
- Mapping to "temperature", τ, and magnetic field, h, of 3d Ising model:

$$\tau = a_1(T-T_c)+a_2(\mu-\mu_c), \quad h = b_1(T-T_c)+b_2(\mu-\mu_c)$$

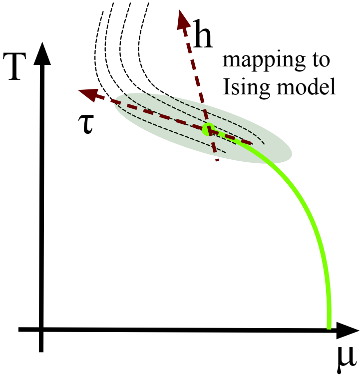


- The mapping is not conformal in general, so the angles are not preserved.
- Universality does not constrain the location of critical point.
- The size of the critical region is also a non-universal property.
- Universal scaling function

$$p_s \propto -b^{-d} f(\tau b^{y_\tau}, h b^{y_h}), \quad y_\tau \approx 1.57, \quad y_h \approx 2.48$$

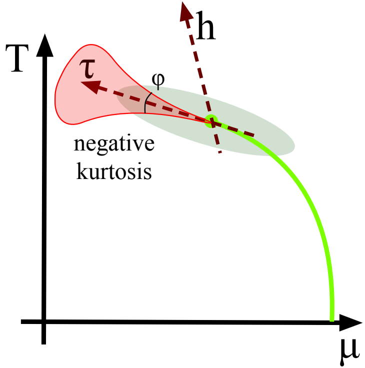
Critical point: properties

- Singular contribution to cumulants $c_n \propto \xi^{3\left(\frac{n\beta\gamma}{2-\alpha}-1\right)}$, where ξ is the correlation length. ξ diverges at critical point.
- $3\left(\frac{n\beta\gamma}{2-\alpha}-1\right)$ is positive for $n \geq 2$, $\frac{\beta\gamma}{2-\alpha} \approx 0.86$
- Divergence of the correlation length dependes on the direction:
  along τ axis: $\xi \propto \tau^{-\nu} \approx \tau^{-0.64}$
  along any other direction: $\xi \propto h^{-\nu/\beta\delta} \approx h^{-0.4}$



- While τ axis direction is not common in general, it is the most relevant for HIC physics. τ coincides with an isentrope, s/n_B = const: consequence of the universality and hierarchy of the critical dimensions $y_\tau < y_h$.
  Indeed, for $h = 0$: $T - T_c \propto \mu - \mu_c$. Therefore, taking derivative of pressure and neglecting corrections proportional to $(T - T_c)^{1-\alpha}, \alpha \approx 0.125, \rightsquigarrow n_B - n_{Bc} = s - s_c = 0$.
- Isentropes in critical regions are parallel to the phase transition line! Good news for HIC!

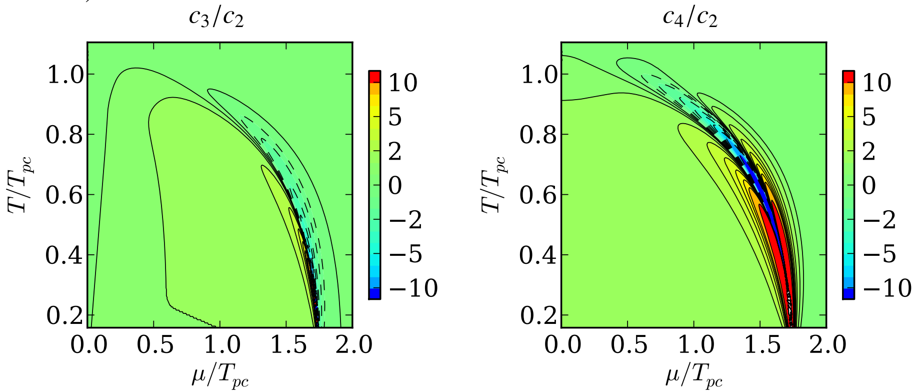
- Careful analysis of the sign shows that kurtosis is negative if approached along the phase transition line (M. Stephanov, 2011)
- Although the angles and relation between minimum and maximum of kurtosis along h=const lines are known, these does not provide any useful information for QCD, because the mapping is not conformal and not universal.
- From the discussion we had before: negative kurtosis is not a unique signature of CP.
- Another important piece of information from universality: below the phase transition region c_3 and c_3/c_2 receives positive (and potentially large) contribution from critical fluctuations.



Schematically:

$$\frac{c_3}{c_2} = \tanh\left(\frac{\mu}{T}\right) + (\text{Positive const}) \times \xi^{2.58}$$

Model calculations: Polyakov loop extended Quark Meson Model (quarks perturbatively interacting with mean-field gluon field, A_0, plus chiral degrees of freedom)

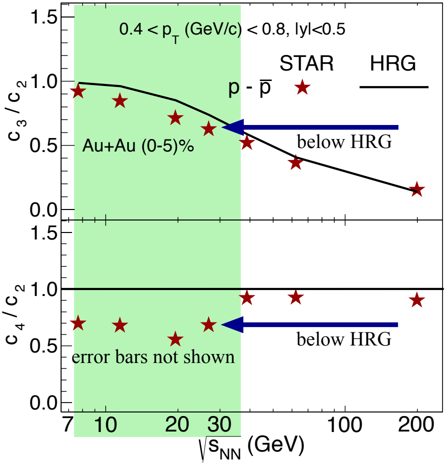


Critical point: experiment

CP "predictions":

- Deviation from HRG at the same energy for c_3/c_2 and c_4/c_2. ✓
- Deviation from HRG stronger for c_4/c_2 than for c_3/c_2. ✓
- c_3/c_2 above HRG, c_4/c_2 below HRG. ✗

Experiment vs HRG



from P. Netrakanti et al, 1405.4617

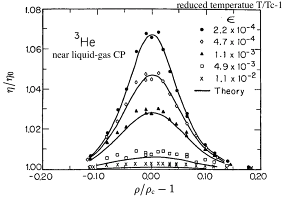
Critical point: dynamics

- Critical phenomena are not limited to statics only.
- Critical slowing down, divergent kinetic coefficients.
- QCD CP is expected to belong to the model H of Hochenberg-Halperin classification.
  - Model H: fluctuations of conserved order parameter (entropy for liquid-gas phase transition) coupled to transverse momentum.
    For QCD dynamics: order parameter is the mixture of the baryon number and the chiral condensate.
  - Dynamical universality classes do not completely characterized by static universality classes! New ingredients: conservation laws, mode-mode couplings.

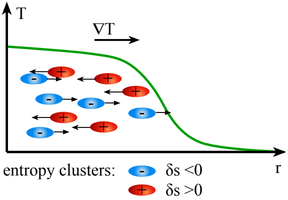
- Divergent bulk viscosity $\eta \propto \xi^{\frac{1}{19}}$
  The divergence is very mild.

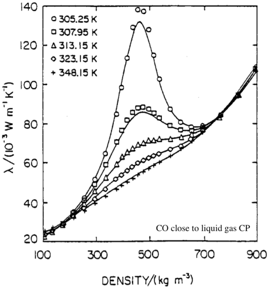
- Divergent bulk viscosity $\zeta \propto \xi^{z-\alpha/\nu} \approx \xi^{2.8}$
  z is the dynamic critical exponent, $z \approx 4 - 18/19$.

- Divergent heat (and charge) conductivity $\lambda \propto \xi^{\frac{18}{19}}$



Clusters with lower entropy tend to move along the temperature gradient; Clusters with higher entropy tend to move opposite to the temperature gradient.





C. Agosta et al J. Low Temp. Phys. 67, 237 (1987). J. Luettmer-Strathmann et al J. Chem. Phys 103, 7482 (1995).

Divergent bulk viscosity

Large bulk viscosity may lead to low effective pressure, lower than pressure of hadronic phase.
$\rightsquigarrow$ hadronic phase will become a proffered phase
$\rightsquigarrow$ intensive bubble formation (cavitation)
$\rightsquigarrow$ instability of fluid evolution

Experimental observables:

- Irregularities in flow and HBT.
- Formation of clusters $\rightsquigarrow$ non-trivial correlation of hadrons (surface tension).
- Hadronization at higher T lower μ compared to HRG predictions; thus anomalies in rations of multiplicities π/p.

$$\zeta \propto \xi^{2.8}$$

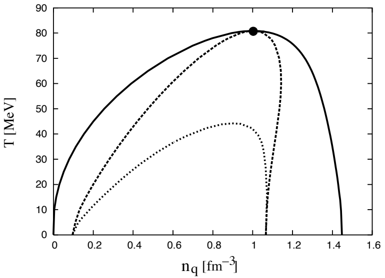
compare to the divergence of the cumulants

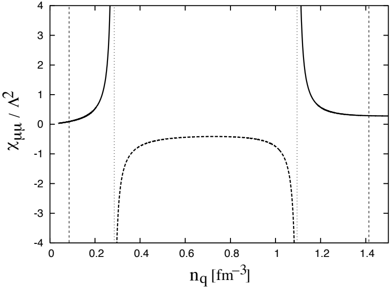
$$c_2 \propto \xi^{2.16} \quad c_3 \propto \xi^{4.74} \quad c_4 \propto \xi^{7.32}$$

- Non-equilibrium statics: divergent c_2 on spinodals
- Divergence is rather mild

$$c_2 \propto |\mu - \mu_0|^{-1/2}$$

In the hadron phase, singular contribution to c_2, c_3 and c_4 is positive.





C. Sasaki, B. Friman, K. Redlich 2007

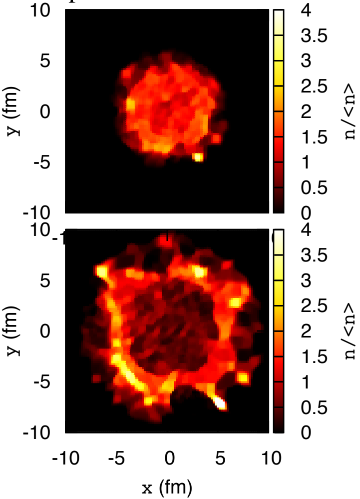
First-order phase transition: dynamics

- Spinodal decomposition: formation of bubbles.
- Similar to cavitation, ratio of the surface tension to the viscosity plays a key role.
- Clusters $\rightsquigarrow$ multi hadron correlations in rapidity.

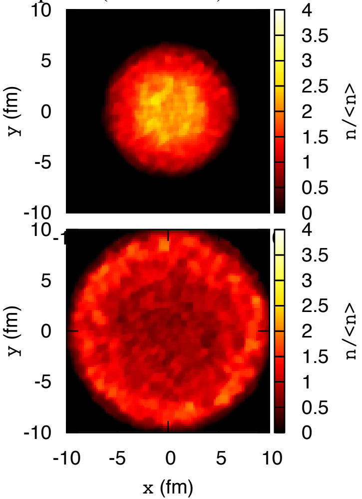
First-order phase transition: chiral model simulations

C. Herold, M. Nahrgang, et al arXiv:1304.5372

First order phase transition



Critical point (ideal limit)



Quarkyonic phase: statics and dynamics

- Quarkionic chiral spiral, Gross-Neveu model with vector interaction.

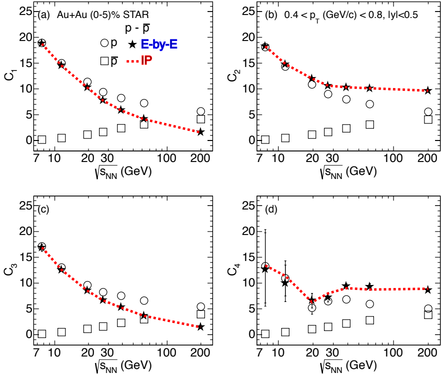
- c_2 is finite, while

$$c_3 \propto \ln(\mu - \mu_0), \quad c_4 \propto \frac{1}{(\mu - \mu_0)}$$

  Very mild divergence in c_3, strong divergence in c_4. Both come with the negative sign!
- Quarkyonic phase is inhomogeneous and characterized by a specific wave number, k.
- Photon spectra will be modified at k!
  G. Torrieri et al Phys.Rev.Lett. 111 (2013) 1, 012301

Experimental data shows that net proton fluctuations can be described by independent proton/antiproton fluctuations

$$c_n^{\text{net}} = c_n^{\text{p}} + (-1)^n c_n^{\bar{\text{p}}}$$



Theory:

- No guidence from lattice QCD on $\langle n_b n_{\bar{b}} \rangle$
- Models with phase transitions: serious limitations.

Independent proton/antiproton II

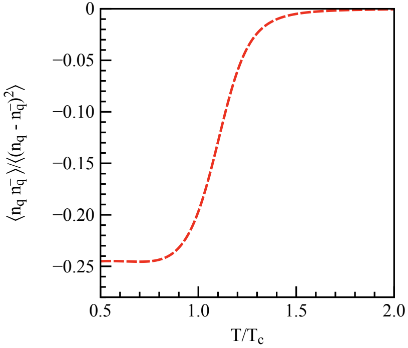
- Model: separate potential for quarks (μ_q) and anti-quarks ($\mu_{\bar{q}}$).
- Confinement is encoded in suppression of quarks at low T.
- After computation, $\mu_q = \mu_{\bar{q}}$, $\langle n_q n_{\bar{q}} \rangle = \left. \frac{\partial^2 p}{\partial \mu_q \partial \mu_{\bar{q}}} \right|_{\mu_q = \mu_{\bar{q}}}$

Possible sources of correlation

- σ-meson exchange (particularly important close to CP)
- Polyakov loop l

Signal is very weak and can be lost owing to hadronization, "acceptance" corrections, etc.

Polyakov loop only:



Conclusions

- QCD phase diagram can be studied with fluctuations! Each transition has something unique.
- However, some unique features demand a very high resolution.
- Can we achieve this resolution in HIC?